Scale resolved intermittency in turbulence

Siegfried Grossmann [1] and Detlef Lohse [1,2]

February 5, 2008

[1] Fachbereich Physik, Philipps-Universität,
Renthof 6, D-35032 Marburg, Germany

[2] The James Franck Institute, The University of Chicago,
5640 South Ellis Avenue, Chicago, IL 60637, USA

The deviations $\delta\zeta_m$ ("intermittency corrections") from classical ("K41") scaling $\zeta_m = m/3$ of the mth moments of the velocity differences in high Reynolds number turbulence are calculated, extending a method to approximately solve the Navier-Stokes equation described earlier. We suggest to introduce the notion of scale resolved intermittency corrections $\delta\zeta_m(p)$, because we find that these $\delta\zeta_m(p)$ are large in the viscous subrange, moderate in the nonuniversal stirring subrange but, surprisingly, extremely small if not zero in the inertial subrange. If ISR intermittency corrections persisted in experiment up to the large Reynolds number limit, our calculation would show, that this could be due to the opening of phase space for larger wave vectors. In the higher order velocity moment $\langle |u(p)|^m \rangle$ the crossover between inertial and viscous subrange is $(10\eta m/2)^{-1}$, thus the inertial subrange is *smaller* for higher moments.
PACS: 47.25.Cg, 47.10.+g, 05.40.+j

Experimentally turbulent flow has long been known to be intermittent [1]. A signal is called intermittent, if there are relatively calm periods which are irregularly interrupted by strong turbulent bursts either in time or in space. Correspondingly, the probability density function (PDF) develops enhanced tails of large fluctuations and a center peak due to the abundance of calm periods, i.e., the PDF becomes of stretched exponential type instead of being Gaussian. This also means that the r-scaling exponents ζ_m of the velocity differences,

$$< |\mathbf{v}_r(\mathbf{x})|^m > \equiv < |\mathbf{u}(\mathbf{x} + \mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t)|^m > \propto r^{\zeta_m}, \tag{1}$$

do not vary linearly with m, namely as m/3, as was originally suggested by dimensional analysis of the universal, inertial subrange of fully developed turbulent flow [2, 3]. Any deviations $\delta\zeta_m = \zeta_m - m/3$ are called intermittency corrections.

Phenomenological intermittency *models* describe the measured intermittency corrections $\delta\zeta_m$ more or less successfully. For a detailed discussion see e.g. [4, 5]. But from our point of view an understanding of intermittency has to come from the *Navier-Stokes equation.*

As full simulations for high Reynolds number ($Re \approx 10^6$) turbulence are out of range even for near future computers, one is thrown on approximations of the Navier-Stokes dynamics.

The main idea of such an approximation has been introduced by us in [5, 6]. Meanwhile we have considerably improved our approach and in this paper we employ it to determine the intermittency corrections $\delta\zeta_m$. For completeness, we briefly repeat how our approximation scheme works.

It starts from the common Fourier series in terms of plane waves $\exp(i\mathbf{p} \cdot \mathbf{x})$, $\mathbf{p} = (p_i)$, $p_i = n_i L^{-1}$, $n_i = 0, \pm 1, \pm 2, \ldots$. The periodicity volume is $(2\pi L)^3$, L is the outer length scale. To deal feasibly with the many scales present in turbulent flow, we only admit a geometrically scaling subset K of wave vectors in the Fourier sum, $K = \cup_l K_l$, thus $u_i(\mathbf{x}, t) = \sum_{\mathbf{p} \in K} u_i(\mathbf{p}, t) \exp(i\mathbf{p} \cdot \mathbf{x})$. Therefore we have called our approximation scheme "Fourier-Weierstrass decomposition" [5, 6]. $K_0 = \{\mathbf{p}_n^{(0)}, n = 1, ..., N\}$ contains appropriately chosen wave vectors, which already have quite different lengths but dynamically interact to a good degree. The $K_l = \{\mathbf{p}_n^{(l)} = 2^l \mathbf{p}_n^{(0)}, \quad n = 1, ..., N\}$, $l = 1, ..., l_{max}$, are scaled replica of K_0 which represent smaller and smaller eddies. l_{max} is chosen large enough to guarantee that the amplitudes $\mathbf{u}(\mathbf{p}_n^{l_{max}}, t)$ of the smallest eddies are practically zero. Of course, l_{max} depends on the viscosity ν and thus on Re.

We solve the Navier-Stokes equation for incompressible flow (i.e., $\mathbf{p} \cdot \mathbf{u}(\mathbf{p}) = 0$) in the subspace defined by the wave vector set K,

$$\dot{u}_i(\mathbf{p}) = -i M_{ijk}(\mathbf{p}) \sum_{\mathbf{q_1}, \mathbf{q_2} \in K, \mathbf{q_1} + \mathbf{q_2} = \mathbf{p}} u_j(\mathbf{q_1}) u_k(\mathbf{q_2}) - \nu \mathbf{p}^2 u_i(\mathbf{p}) + f_i(\mathbf{p}). \tag{2}$$

The set K_0 is chosen in a way that as many triadic Navier-Stokes interactions $\mathbf{p} = \mathbf{q_1} + \mathbf{q_2}$ as possible are admitted. The degree of the nonlocality in p-space of any triadic interaction can be characterized by the quantity $s :=$

$max(p, q_1, q_2)/min(p, q_1, q_2)$. We allow for s up to 5.74. To force the flow permanently we choose

$$\mathbf{f}(\mathbf{p}, t) = \frac{\epsilon \, \mathbf{u}(\mathbf{p}, t)}{\sum_{\mathbf{q} \in K_{in}} |\mathbf{u}(\mathbf{q}, t)|^2} \quad \text{for} \quad \mathbf{p} \in K_{in}, \tag{3}$$
$$\mathbf{f}(\mathbf{p}, t) = 0 \quad \text{for} \quad \mathbf{p} \notin K_{in},$$

as a deterministic, non-stochastic driving. $K_{in} \subset K_0$ only contains the wave vectors with the three smallest lengths. The corresponding amplitudes $\mathbf{u}(\mathbf{p}, t)$ carry the largest energy.

For the numerical calculations the times are measured in units of the largest eddies turnover time $L^{2/3}\epsilon^{-1/3}$ and the velocities in units of $(L\epsilon)^{1/3}$. The Reynolds number can then be defined by $Re = \nu^{-1}$. The coupled set of $(3 \cdot (l_{max} + 1) \cdot N)$ equations (2) is integrated with the Burlirsch-Stoer integration scheme with adaptive stepsize. All averages are time averages, denoted by $\langle \dots \rangle$.

We remark that the density of the admitted wave vectors per p-interval decreases as $1/p$ in our reduced waveset approximation, whereas it increases as p^2 in full grid simulations, see Fig.1. But this shortcoming at the same time is the main advantage of our approximation, because many more scales than in full simulations can be taken into account. In [6] and [7] we achieved $Re = 2 \cdot 10^6$, i.e., 3 decades. We used $N = 26$, $l_{max} = 10$ and $N = 80$, $l_{max} = 12$, respectively. The main features of fully developed turbulence as chaotic signals, scaling, turbulent diffusion, etc., are well described within our approximation [5, 6]. In particular, our solutions show *small scale intermittency*. This is accounted for by a competition effect between turbulent energy transfer downscale and viscous dissipation [5].

The main improvements of our new treatment in the present paper are: (i) The number of modes per K_l is considerably increased up to N=86 instead of N=38 in [5] or N=26 in [6]. Thus the number of contributing triades in eq.(2) is much larger, see table 1. (ii) We now also allow for *nonlocal* interactions in p-space. The nonlocality of the waveset K with respect to the Navier-Stokes dynamics (2) can be quantified by s_{max}, defined as the maximum of the s-values of all contributing triades. In our former calculations [5, 6] we had $s_{max} \leq 2$, which means that eddies can at most decay in half size eddies, whereas now for $s_{max} \approx 6$ (see table 1) sweeping of small eddies on larger ones (up to a factor of 6) is possible. (iii) We can now consider the individual $\mathbf{u}(\mathbf{p}, t)$ instead of the whole shells $\mathbf{u}^{(l)} = \sum_{\mathbf{p} \in K_l} \mathbf{u}(\mathbf{p})$, what had to be done in [5, 6] because of the smaller number of triadic interactions. (iv) We are now much beyond a shell model [5, 6, 8], since on the $|\mathbf{p}|$-axis the elements of the wave vector subsets K_l interpenetrate and intermingle considerably.

We now offer our results.

The spectra $\langle |\mathbf{u}(\mathbf{p})|^2 \rangle$ and $\langle |\mathbf{u}(\mathbf{p})|^6 \rangle$ calculated with N=86 wave vectors in K_0 and with $Re = 125\ 000$ are shown in Fig.2a. For comparison the same spectra are also given for N=38 as used in our previous work (Fig.2b, [5]). As expected the scatter becomes less with increasing N. We fit the spectra with the three parameter

functions

$$\langle |\mathbf{u}(\mathbf{p})|^m \rangle = c_m p^{-\zeta_m} \exp\left(-p/p_{D,m}\right). \tag{4}$$

In Table 2 the fit parameters ζ_m and $p_{D,m}$ are listed. The ansatz (4) is theoretically known [9] to hold for m=2. We find that it also holds for $m > 2$ with $p_{D,m} = 2p_{D,2}/m$ as one can expect, if in the VSR the higher moments factorize. For the dissipative cutoff $p_D := p_{D,2}$ we obtain $p_D = (11\eta)^{-1}$, where $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov length. This well agrees with the long known (experimentally [10] and theoretically [11, 12]) crossover between the viscous subrange VSR and the inertial subrange ISR in the structure function $D^{(2)}(r) = \langle |\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})|^2 \rangle$ at r about 10η . According to our finding $p_{D,m} = 2p_{D,2}/m$ the crossover $p_{D,m}$ in higher order moments $\langle |u(p)|^m \rangle$ occurs at smaller p, namely, approximately at $(10\eta m/2)^{-1}$. The ISR for higher moments is thus definitely smaller. This does *not* necessarily mean that the ISR for higher order *structure* functions $D^m(r)$ is also smaller, because they are not simply connected with $\langle |u(p)|^m \rangle$ via a Fourier transform as in the case $m = 2$. In passing by we remark that by properly renorming the wave vector p and the spectral intensity, the spectra (4) can be shown to the *universal* for all Reynolds numbers both in experiment [13] and in full simulations [14] and in our approximate Navier Stokes solution [7].

The intermittency corrections $\delta\zeta_m$ from our overall fit (4) are much smaller than the experimental ones around. At the other hand we observe here as in [5] that there *is* much intermittency in the signals, at least for small scales. We therefore determined the exponents ζ_m in (4) by fitting restricted p-ranges only. We suggest to introduce "local" $\zeta_m(p)$. These are defined by local fits of the type (4), using for each wave vector $\mathbf{p}$ the moments in the local p-decades $[p/\sqrt{10}, p\sqrt{10}]$. The cutoff wave vectors are kept fixed at their global values $p_D = (11\eta)^{-1}$, $p_{D,m} = 2p_D/m$. Also, as before (see caption of table 2) we devide the local $\zeta_m(p)$ by $\zeta_3(p)$.[1]

The astonishing results are shown in Fig.3a: There are large intermittency corrections $\delta\zeta_m(p)$ for the small scales (large p, VSR), only moderate intermittency corrections for the large scales (small p, stirring subrange SSR), but hardly any deviations for p in the ISR.

The small scale intermittency is well understood [15, 16] and was extensively discussed in [5]. It is best seen in small scale quantities as for example in the energy dissipation rate $\epsilon(\mathbf{x}, t)$ or in the vorticity. Here we observe in addition that the intermittency corrections $\delta\zeta_m(p)$ in the VSR remarkably well agree with the r-scaling exponents $\mu(m/3)$, defined by $\langle \epsilon_r^{m/3} \rangle \propto r^{-\mu(m/3)}$, which we had already calculated in [5]. This means that for r in the VSR Kolmogorov's refined similarity hypothesis (RSH) $v_r \propto (\epsilon_r r)^{1/3}$ seems to be well fulfilled. This also is in agreement with Kraichnan's [17], Frisch's [18], and our [19] objections against the RSH in the ISR, arguing that for r in the ISR v_r is an ISR quantity, whereas ϵ_r still mainly is a

---

[1]Instead of defining $\delta\zeta_m(p) = \zeta_m(p)/\zeta_3(p) - m/3$ one could also take the deviation $\tilde{\delta}\zeta_m(p)$ of the $\zeta_m(p)$ from the linear behaviour as a measure of intermittency. It holds $\tilde{\delta}\zeta_m(p) \equiv \zeta_m(p) - m\zeta_3(p)/3 = \zeta_3(p)\delta\zeta_m(p)$. In the VSR it is $\zeta_3(p) \approx 1$, so both definitions for the intermittency corrections essentially agree, but in the VSR we find $\zeta_3(p) > 1$, so $\tilde{\delta}\zeta_m(p) > \delta\zeta_m(p)$.

VSR quantity. Thus a relation like the RSH should only be expected, if r is in the VSR and both v_r and ϵ_r are VSR quantities.

Our result is also consistent with latest full numerical simulations [20], which find the RSH fulfilled. Note that in these simulations r is always in or at least near the VSR since Re is still small. And last not least our finding also agrees with the observation of Chen et.al. [21] that the RSH is less and less fulfilled the larger r becomes. For further comparison with experiment, see below.

Before we interpret the behaviour of the $\delta\zeta_m(p)$ in the stirring subrange SSR and in the ISR, we checked how our findings depend on various changes of our Navier-Stokes approximation: (i) To be sure that the SSR-intermittency does not depend on the kind of forcing (3), we compared with the alternative forcing $\mathbf{f}(\mathbf{p}) \propto \mathbf{u}(\mathbf{p})$, again $\mathbf{p} \in K_{in}$. We also took a random forcing, but the results did not change noticeably. (ii) We varied the set K_{in} and allowed for more or for fewer modes which are stirred, but again there was no change. (iii) We varied the type of wave vectors in K_0 and their number N as well as the maximal nonlocality s_{max} of the contributing triadic interactions (see table 1). Again, no sizeable change. In particular, the intermittency corrections did not increase with increasing nonlocality of the triadic interactions as we speculated in [5]. (iv) Different values of $\delta\zeta_m(p)$ were only obtained when the flow field was not yet statistically stationary, see Fig.4. In Fig.4a we averaged over 7 large eddy turnover times only. The total rate of dissipated energy $\epsilon_{diss} = \nu \sum_{\mathbf{p} \in K} \mathbf{p}^2 \langle |\mathbf{u}(\mathbf{p})|^2 \rangle$ still exceeds the constant input ϵ by about 1%. In this case there are considerable intermittency corrections $\delta\zeta_m(p)$ for all p, which go down drastically in the ISR if one averages over 70 large eddy turnover times, see Fig.4b, where we had statistically stationary results. (Stationarity is identified from the balance between the total dissipation rate and the total input rate.) Similar observations have been made when analysing experimental signals [22]. (v) To demonstrate how $\delta\zeta_m(p)$ varies from run to run we refer to Fig.5. The deviations $\delta\zeta_m(p)$ for p in the ISR are very small, but still seem to be significant. (vi) We decreased the degree of locality of the $\zeta_m(p)$ by fitting the larger range $[p/\sqrt{20}, p\sqrt{20}]$, see Fig.3b. Again no qualitative change; $\delta\zeta_m(p)$ now tends to become even smaller in the ISR. (vii) We artificially extended the ISR by putting $\nu = 0$ and extracting the energy from the smallest eddies by using a phenomenological eddy viscosity as employed in [6]. Now, as expected, $\delta\zeta_m(p) \approx 0$ also for the large p, i.e., the small scale intermittency really originates from the competition between transport downscale and the viscous damping. (viii) One might speculate that intermittency corrections in the ISR would occur if our Fourier-Weierstrass ansatz would not only be wave number but also space resolving as in [23]. But when doing this we found that the intermittency corrections observed in [23] vanish if the number N of wave vectors in K_0 is increased [5]. One should note that we include in fact some degree of position space localization since any Fourier representation with many modes already allows for localization in space.

To have another check, we also calculated the scale dependent flatness $F(p) = \langle |\mathbf{u}(p)|^4 \rangle / \langle |\mathbf{u}(p)|^2 \rangle^2 \propto p^{-\zeta_4 + 2\zeta_2}$. If there is intermittency, then $2\zeta_2 > \zeta_4$, thus $F(p)$

has to increase with p. In fact we find such an increase of $F(p)$ in the SSR from $F(p = 3) \approx 2.7$ (< 3, a result achieved also in various full numerical simulations and experiments, see e.g. [14] for a recent reference.) to the value $F(p) \approx 3.0$ valid for a Gaussian distribution. For p in the ISR $F(p) = 3$ stays constant. Approaching the VSR by further increased p, the flatness now strongly grows [5]. This can be understood as being due to the small scale intermittency, as we extensively reported in [5]. This behaviour of $F(p)$ well agrees with the above described findings for $\delta\zeta(p)$.

Finally we report how the flatness of the velocity derivative $F_{1,1} = \langle(\partial_1 u_1)^4\rangle / \langle(\partial_1 u_1)^2\rangle$ behaves as a function of Re. For the large Reynolds numbers which we consider we find $F_{1,1} = 3.15$, independent of Re. This again means, we find no intermittency as models which are constructed to describe intermittency obtain an increase of $F_{1,1}$ with the Re number in terms of the ISR intermittency exponent $\mu(2)$, $F_{1,1} \propto Re^{3\mu(2)/4}$, see e.g. [24, 19].

Two conclusions of our findings are possible.

First, the very small if not missing intermittency might be due to our approximation. Even in the present, considerably improved ansatz the larger wave vectors are still thinned out, cf.Fig.1. If this argument was valid, the ISR- intermittency would have been identified by us as an effect of the opening of the phase space for larger wave vectors. Consequently, there should be no intermittency in 2D turbulence, where the energy cascade is inverse – and infact Smith and Yakhot [25] do not find intermittency in numerical 2D turbulence.

The second possible conclusion is that there indeed might be no intermittency in the pure ISR in the limit of large Re. Of course, if so that must be due to the particular form of the nonlinearity, namely the $\mathbf{u} \cdot grad\mathbf{u}$-term in the Navier-Stokes equation. It provides energy transport both downscale and upscale which, as our solutions show, fluctuates wildly and with large amplitudes around a rather small mean value of downscale transport. This nearly symmetric down- and up- scale transport is broken on the large scales (i.e., in the SSR) due to the finite size of the system. The largest eddies do not get energy by turbulent transfer downscale but only deliver turbulent energy to smaller scales. The symmetry of transport is also broken for small scales by the competition with the viscous dissipation. May be that the symmetry breaking mechanisms cause the large and the small scale intermittency. Note that Galileian invariance is only broken by the boundaries, i.e., by the finite size of the system. Both dissipation and our forcing scheme keep it.

This second possible conclusion is in agreement with a recent theory developed by Castaing et.al. [26]. This theory predicts that $\delta\zeta_m = 0$ in the limit of large Re, and that in this limit $F_{1,1}$ is independent of Re, which we find, as mentioned above. The value of the $F_{1,1}$-limit, if it exists, is probably larger than what we find. Vincent and Meneguzzi [27] calculate already $F_{1,1} = 5.9$ for a Taylor Reynolds number $Re_\lambda \approx 100$. Castaing et al. [26] find from their data analysis, that the flatness $F(p)$ increases as $\log F(p) \propto (\eta p)^\beta$ with $\beta \propto 1/\log(Re_\lambda/75)$. Our finding $F(p) \approx 3$ in the ISR (for $Re \approx 10^5$, $Re_\lambda \approx 9000$) well agrees with the large Re

limit of this experimental behavior. Note that in experiment it is still $\beta \approx 0.24$ even for $Re_\lambda = 2720$ [26]. Also the measured intermittency correction $\delta\zeta_m$ at least for $m \geq 6$ are not 0 even for $Re_\lambda = 2720$ [26, 28]. However it could well be that in experiment the intermittency corrections might be overestimated, because they will tend to increase if the averaging time is not large enough and the flow is not yet statistically stationary, see above, Fig.4, and the remarks in Ref. [22].

Our finding, that there might be three ranges for high Re turbulence – namely, the SSR with moderate intermittency, the ISR with practically no intermittency, and the VSR with strong intermittency – might also be supported by some experimental data arround. In Fig.6 the spectrum $\langle|\mathbf{u}(\mathbf{p})|^2\rangle$ is shown, taken from Gagne's wind tunnel measurements [29] with the very high Reynolds number $Re_\lambda = 2720$. While in the ISR $\zeta_2 = 0.67$, i.e., $\delta\zeta_2 = 0$, seems to be a good fit, for the large scales (SSR) the exponent $\zeta_2 = 0.70$ is more appropriate. In the VSR the exponential damping according to (4) is not separated, so that ζ_2 cannot reliably be identified in that range. Note, that in the same experiment higher moments and the scale resolved flatness $F(p)$, which is more sensitive to intermittency corrections, show intermittency *also* in the ISR.

A similar interpretation seems possible by inspection of Praskovsky's [30] data for $\langle|\mathbf{v}_r|^6\rangle$ measured at the also very high $Re_\lambda = 3200$, see Fig.7: In the middle of the ISR we clearly have $\delta\zeta_6(p) = 0$, whereas in the VSR it takes the value $\delta\zeta_6 = 0.31$. This is precisely what one expects from the RSH, namely $\delta\zeta_6 \approx \mu \approx 0.30$. In the SSR the intermittency correction is $\delta\zeta_6(p) = 0.27$, which is considerably larger than what we found in this range. May be this is due to the plumes, swirls, or other structures [31] which detach from the boundary in real flow and might increase the intermittency in the SSR.

Finally, we remark that also the quasi-Lagrangian perturbation analysis of the Navier-Stokes equation, done by Belinicher and L'vov [32], leads to $\delta\zeta_m(p) = 0$ for p in the ISR in the large Re limit.

To summarize, it can not yet be ultimatively decided which of the discussed conclusions of our numerical data will turn out to be robust. Either there in fact *is* ISR-intermittency also for $Re \to \infty$ as an effect of phase space opening for large wave vectors (which by construction of our approximation scheme we miss), or there is indeed no ISR-intermittency in the limit of large Re [26, 32]. To decide this alternative, it would be very helpful to at least allow *some* opening of phase space, e.g., to increase the number of wave vectors per level as $\log k$ as already done in a 2D approximate solution of the Navier Stokes equation [33]. If then intermittency does not show up again, we clearly have to favour the conclusion that there is no ISR-intermittency in the large Re limit as our results demonstrate.

References

[1] G. K. Batchelor and A. A. Townsend, Proc. R. Soc. London A **199**, 238 (1949).

[2] A. N. Kolmogorov, CR. Acad. Sci. USSR. **30**, 299 (1941).

[3] C. F. von Weizsäcker, Z. Phys. **124**, 614 (1948); W. Heisenberg, Z. Phys. **124**, 628 (1948); L. Onsager, Phys. Rev. **68**, 286 (1945).

[4] Ch. Meneveau and K. R. Sreenivasan, J. Fluid Mech. **224**, 429 (1991).

[5] S. Grossmann and D. Lohse, Z. Phys. B **89**, 11 (1992); Physica A **194**, 519 (1993).

[6] J. Eggers and S. Grossmann, Phys. Fluids A **3**, 1958 (1991).

[7] S. Grossmann and D. Lohse, Universality in turbulence, Preprint, Marburg, 1993.

[8] E. D. Siggia, Phys. Rev. A **15**, 1730 (1977); R. M. Kerr and E. D. Siggia, J. Stat. Phys. **19**, 543 (1978); M. H. Jensen, G. Paladin, and A. Vulpiani, Phys. Rev. A **43**, 798 (1991).

[9] C. Foias, O. Manley, and L. Sirovich, Phys. Fluids A **2**, 464 (1990).

[10] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* (The MIT Press, Cambridge, Massachusetts, 1975).

[11] H. Effinger and S. Grossmann, Z. Phys. B **66**, 289 (1987).

[12] S. Grossmann and D. Lohse, Phys. Lett. A **173**, 58 (1993).

[13] Z. S. She and E. Jackson, Phys. Fluids A **5**, 1526 (1993).

[14] Z. S. She *et al.*, Phys. Rev. Lett. **70**, 3251 (1993).

[15] R. Kraichnan, Phys. Fluids **10**, 2080 (1967).

[16] U. Frisch and R. Morf, Phys. Rev. A **23**, 2673 (1981).

[17] R. Kraichnan, J. Fluid Mech. **62**, 305 (1974).

[18] U. Frisch, Proc. R. Soc. London A **434**, 89 (1991).

[19] S. Grossmann and D. Lohse, Europhys. Lett. **21**, 201 (1993).

[20] I. Hosokawa and K. Yamamoto, Phys. Fluids A **4**, 457 (1992); J. Phys. Soc. Japan. **60**, 1852 (1991) and addendum J. Phys. Soc. Japan. **62**, 380 (1993); J. Phys. Soc. Japan. **62**, 10 (1993).

[21] S. Chen, G. D. Doolen, R. H. Kraichnan, and Z. S. She, Phys. Fluids A **5**, 458 (1993).

[22] W. van de Water, B. van der Vorst, and E. van de Wetering, Europhys. Lett. **16**, 443 (1991).

[23] J. Eggers and S. Grossmann, Phys. Lett. A **156**, 444 (1991).

[24] Ch. Meneveau and K. R. Sreenivasan, Nucl. Phys. B (Proc. Suppl) **2**, 49 (1987); M. Nelkin, Phys. Rev. A **42**, 7226 (1990); J. Eggers and S. Grossmann, Phys. Lett. A **153**, 12 (1991).

[25] L. M. Smith and V. Yakhot, Phys. Rev. Lett. **71**, 352 (1993).

[26] B. Castaing, Y. Gagne, and E. J. Hopfinger, Physica D **46**, 177 (1990); B. Castaing, J. Phys. **50**, 147 (1989); B. Castaing, Y. Gagne, and M. Marchand, Physica D (1993), to appear.

[27] A. Vincent and M. Meneguzzi, J. Fluid Mech. **225**, 1 (1991).

[28] F. Anselmet, Y. Gagne, E. J. Hopfinger, and R.A. Antonia, J. Fluid Mech. **140**, 63 (1984).

[29] Y. Gagne, Thesis, University of Grenoble, 1987.

[30] A. A. Praskovsky, Phys. Fluids A **4**, 2589 (1992).

[31] G. Zocchi, E. Moses, and A. Libchaber, Physica A **166**, 387 (1990).

[32] V. I. Belinicher and V. S. L'vov, Sov. Phys. JETP **66**, 303 (1987).

[33] E. Vazquez-Semadeni and J. H. Scalo, Phys. Fluids A **4**, 2833 (1992).

[34] A. N. Kolmogorov, J. Fluid Mech. **13**, 82 (1962).

[35] U. Frisch, P. L. Sulem, and M. Nelkin, J. Fluid Mech. **87**, 719 (1978).

[36] K. R. Sreenivasan and P. Kailasnath, Phys. Fluids A **5**, 512 (1993).

Table 1

N	interaction number of triades	s_{max}	b
26 [6]	39	1.92	300
38 [5, 6]	102	1.92	170
50	273	3.46	80
74	741	5.00	70
74	729	5.74	70
74	858	3.46	70
80	783	5.74	70
86	966	5.74	65

Characteristic parameters of several different wavesets K_0. N denotes the number of wave vectors in K_0. In the second column the number of triadic interactions $\mathbf{p} = \mathbf{q}_1 + \mathbf{q}_2$ between the wave vectors of any one set K_l is given, l not too large or small to avoid edge effects. s_{max} is the maximal nonlocality (definition see text) of the triadic interactions. b is the dimensionless constant in the structure function $D(r) = \langle |\mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x})|^2 \rangle = b(\epsilon r)^{2/3}$, $r \in$ ISR. The experimental value is b=8.4 [10]. The larger values in our approximation can well be understood [6], the decrease of b with the increase of N is in keeping with that explanation.

Table 2

	$\langle|\mathbf{u}|^2\rangle$	$\langle|\mathbf{u}|^3\rangle$	$\langle|\mathbf{u}|^4\rangle$	$\langle|\mathbf{u}|^6\rangle$	$\langle|\mathbf{u}|^8\rangle$	$\langle|\mathbf{u}|^{10}\rangle$
ζ_m	0.682	1.021	1.359	2.034	2.707	3.370
ζ_m/ζ_3	0.668	1.000	1.331	1.993	2.651	3.301
$\delta\zeta_m$	0.002	0.000	0.002	-0.007	-0.016	-0.033
$\delta\zeta_m(K62)$	0.022	0.000	-0.044	-0.200	-0.444	-0.778
$\delta\zeta_m(ran-\beta)$	0.038	0.000	-0.046	-0.170	-0.347	-0.590
$p_{D,m}$	597	398	299	201	152	122
$2p_{D,2}/m$	597	398	299	199	149	119

Results from the fit (4) to the spectra $\langle|\mathbf{u}(\mathbf{p})|^m\rangle$ obtained with N=86 wave vectors in K_0 for moments up to m=10. $\nu = 8 \cdot 10^{-6}$. The average is over 80 large eddy turnover times (skipping the first 100 turnovers). We fitted the p-range $[0, 1000]$. In general ζ_3 is rather near, but not exactly 1 as it should be according to Kolmogorov's structure equation [10]. We therefore calculate the intermittency corrections from renormalized exponents ζ_m/ζ_3, namely, $\delta\zeta_m = \zeta_m/\zeta_3 - m/3$. For comparison, the values for Kolmogorov's log normal model [34], $\delta\zeta_m = -\mu m(m-3)/18$, are also given, which are well known to fit the data for $m \leq 10$ with $\mu = \mu_2 \approx 0.20$. $\delta\zeta_m(ran-\beta) = -\log_2[1 - x + x(1/2)^{1-m/3}]$ are the intermittency corrections due to the random β-model (x=0.125) [35]. $p_{D,m}$ is the dissipative cutoff, which agrees very well with $2p_{D,2}/m$ as shown in the last row.

Figure captions

Figure 1
Comparison of the subset of wave numbers admitted in the Fourier-Weierstrass decomposition (upper) with the complete p- spectrum (lower), either $0 \leq |\mathbf{p}| \leq 25$ or $0 \leq |\mathbf{p}| \leq 500$. While the exact density of wave numbers increases as p^2, the geometric scaling $2^l K_0$ makes the density decrease, i.e., the smaller scales are less well resolved.

Figure 2
Spectra $\langle|\mathbf{u}(\mathbf{p})|^m\rangle$ for m=2 ($\diamond$) and m=6 (+).
a) N=86 wave vectors in K_0, $\nu = 8 \cdot 10^{-6}$, $s_{max} = 5.74$, $Re_\lambda = 9030$, averaging time from 100 to 180. The input set K_{in} of the forcing (3) consists of the 12 shortest wave vectors. The lines are the fits (4).
b) N=38, $\nu = 8 \cdot 10^{-6}$, averaging time 150 large eddy turnovers, same forcing as in (a).

Figure 3
$\delta\zeta_m(p)$ for m=2,4,6,8,10, bottom to top. Same data as in Fig.2a. The shaded ranges on the right show the Kolmogorov values [34] $\delta\zeta_m = -\mu m(m-3)/18$ for $\mu = 0.20$ through $\mu = 0.30$, because in [36] $\mu = 0.25 \pm 0.05$ is given as "best estimate". In (a) the fit range is $[p/\sqrt{10}, p\sqrt{10}]$, in (b) the larger local range $[p/\sqrt{20}, p\sqrt{20}]$ is chosen.

Figure 4
$\delta\zeta_m(p)$ for m=2,4,6,8,10 (bottom to top). N=80, $\nu = 5 \cdot 10^{-6}$, $s_{max} = 5.74$, $Re_\lambda = 13\,550$. The averaging times are (a) 7 and (b) 70 large eddy turnovers, respectively.

Figure 5
(a) $\delta\zeta_6(p)$ and (b) $\delta\zeta_2(p)$ for N=80, $\nu = 5 \cdot 10^{-6}$, $s_{max} = 5.74$ for different runs. For two runs the averaging time is about 30 large eddy turnovers, for another two runs it is about 70. The (weighed) means and the standard deviations are marked by a dia- mond and by error bars, respectively.

Figure 6
Spectrum $E(r/\eta)$ from [16] for $Re_\lambda = 2720$. Three ranges can be identified. On the large scales the SSR with $\zeta_2 + 1 = 1.70$, i.e., some intermittency corrections, for moderate r the ISR with $\zeta_2 + 1 = 5/3$, i.e., no intermittency, and for small r the VSR.

Figure 7
Moments of the velocity differences $\langle|v_r|^6\rangle/r^2$ ($\circ$) and of the energy dissipation rate $\langle\epsilon_r^2\rangle$ ($\bullet$) (arb.units) against r, taken from [27]. The slopes of these quantities are $\delta\zeta_6(p)$ and $\mu = \mu_2$, respectively. Clearly, for $\langle|v_r|^6\rangle/r^2$ there are three ranges VSR, ISR, and SSR, whereas the VSR quantity $\langle\epsilon_r^2\rangle$ does not show different ranges. The arrows correspond to $20\eta/L$ and $1/5$, respectively.